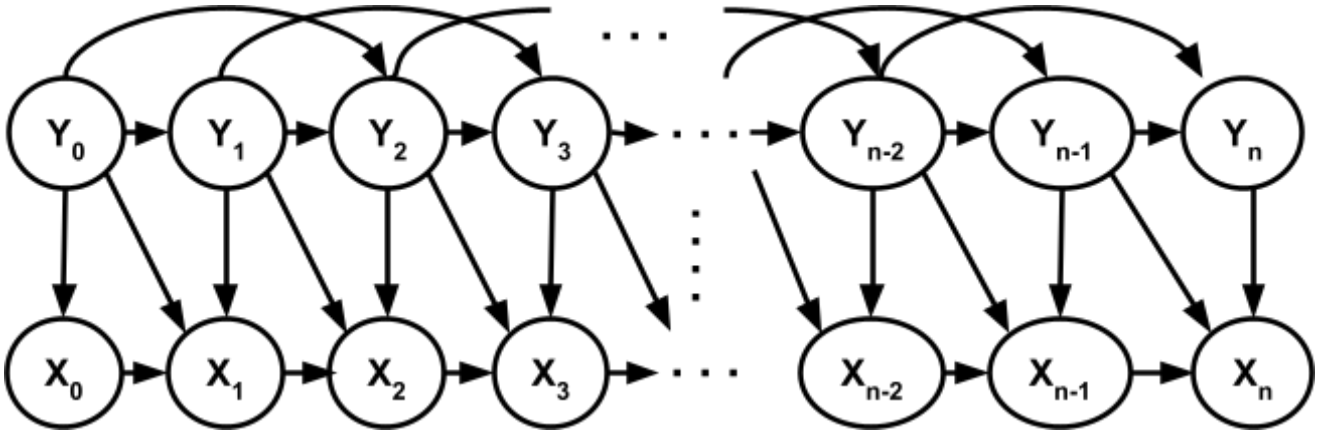
Question 1 - Bayesian Networks

Consider the Bayes Network below.

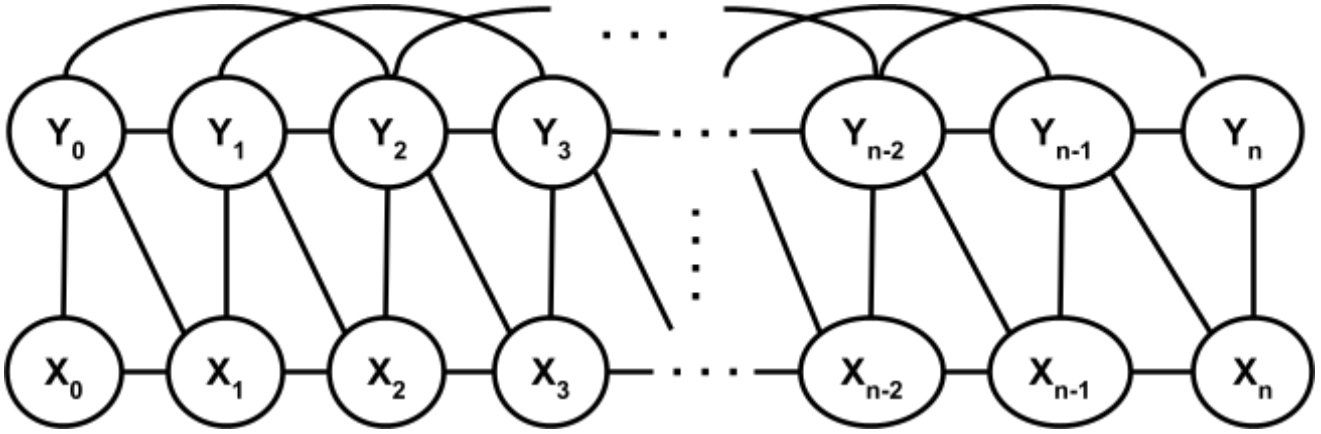


A) Write the joint distribution as the product of CPDs.

B) Which of the following statements are **True**, and which are **False** (in general). For each statement, write the word "True" or an Active Trail rejecting the statement.

	True/Active Trail		True/Active Trail
$Y_0 \perp Y_3 \mid Y_1$		$X_1 \perp X_3 \mid X_2, Y_1, Y_3$	
$Y_0 \perp Y_3 \mid Y_1, Y_2$		$X_0 \perp X_n \mid X_m, Y_m \ (0 < m < n)$	
$X_1 \perp X_3 \mid X_2, Y_2$		$X_0 \perp X_n \mid X_m, Y_{m-1}, Y_m \ (1 < m < n)$	
$X_1 \perp X_3 \mid X_2, Y_2, Y_3$		$X_0 \perp X_n \mid X_m, Y_{m-2}, Y_m \ (2 < m < n)$	

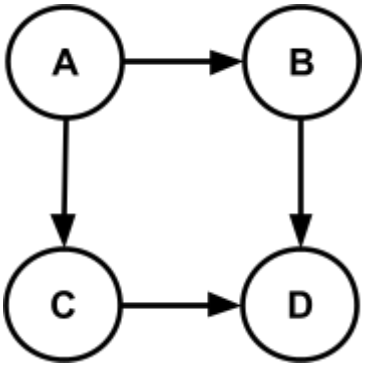
C) Consider the following MRF. Write the joint distribution in the most general form (in terms of the potentials over cliques.)



D) Repeat part (B) for the Markov Network above.

	True/Active Trail		True/Active Trail
$Y_0 \perp Y_3 \mid Y_1$		$X_1 \perp X_3 \mid X_2, Y_1, Y_3$	
$Y_0 \perp Y_3 \mid Y_1, Y_2$		$X_0 \perp X_n \mid X_m, Y_m \ (0 < m < n)$	
$X_1 \perp X_3 \mid X_2, Y_2$		$X_0 \perp X_n \mid X_m, Y_{m-1}, Y_m \ (1 < m < n)$	
$X_1 \perp X_3 \mid X_2, Y_2, Y_3$		$X_0 \perp X_n \mid X_m, Y_{m-2}, Y_m \ (2 < m < n)$	

Question 2

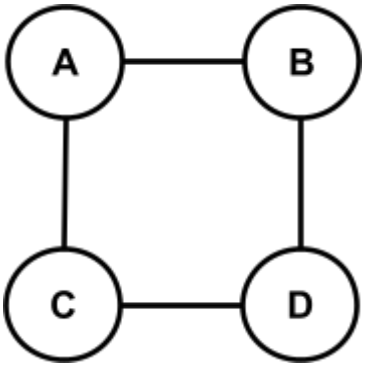


Consider the following Bayes Net on binary variables $A, B, C, D \in \{0, 1\}$, with CPDs defined as:

$P(A) = 3/4 - A/2$
$P(B \mid A) = (AB - B + 1) / (A + 1)$
$P(C \mid A) = (1 - CA) / (2 - A)$
$P(D \mid B, C) = (1 - D + DBC) / (1 + BC)$

A) Derive $P(D \mid A)$. Simplify your answer as much as you can.

B) Derive $P(D \mid A)$ for the following MRF on binary variables $A, B, C, D \in \{0, 1\}$,and with the joint distribution below. Simplify as much as possible.



$$P(A, B, C, D) = \frac{1}{Z} exp\,(AB + 1(B = D) + \min(A, C) + \max(C, D))$$

C) Derive the partition function $Z(A, B)$ for the CRF below on binary variables $A, B, C, D \in \{0, 1\}$.

$$P(C, D \mid A, B) = \frac{1}{Z(A,B)} exp\,(AB + 1(B = D) + \min(A, C) + \max(C, D))$$